The Management University of Africa

Sponsored by the Kenya Institute of Management

UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF ARTS EDUCATION

MTH 211 : LINEAR ALGEBRA

DATE: 31ST March 2022

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provided.**

QUESTION ONE

a) Solve the matrix equation for , or explain why no solution exists;

$$\alpha\begin{bmatrix} 6 & 3 & 2 \\ -7 & 0 & -4 \end{bmatrix} + \begin{bmatrix} 7 & -3 & 1 \\ 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 19 & 12 & 6 \\ 6 & 4 & -2 \end{bmatrix}$$ **(3 marks)**

b) Compute $|2r + 3s|$ given that $r = 2i - 4j + 5k$ and $s = 3i + 7j - k$ **(4 marks)**

c) Evaluate the following and express as a vector; **(3 marks)**

$$2\begin{bmatrix} -2 \\ -11 \\ 13 \\ 10 \end{bmatrix} + 4\begin{bmatrix} 7 \\ 9 \\ -2 \\ 6 \end{bmatrix} =$$

d) Solve the following system of equations;

$$7x + 2y - z = 4$$
$$x + 3y + 2z = 1$$
$$2x - 4y - z = 12$$ **(5 marks)**

e) The points A, B, C and D have position vectors $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 10 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ respectively, relative to an origin O. A point E divides DC internally in the ratio 2:3. Another point G divides BC externally.

i. Prove that the points O, A and E are collinear **(4 marks)**

ii. Compute the coordinated of point G is the point A, E and G are collinear **(6 marks)**

QUESTION TWO

a) Evaluate the value of α and β that solve the following equation;

$$\alpha\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} + \beta\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 4 & 1 \end{bmatrix}$$ **(3 marks)**

b) Compute the product MN in the matrices below;

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & -6 & -3 \end{bmatrix} \qquad N = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ **(4 marks)**

c) Let T be the set of columns in a matrix B below. Define $W = \langle T \rangle$ and find a set R so that R has 3 vectors, R is a subset of T and $W = \langle R \rangle$. **(8 marks)**

$$\begin{bmatrix} -3 & 1 & -2 & 7 \\ -1 & 2 & 1 & 4 \\ 1 & 1 & 2 & -1 \end{bmatrix}$$

QUESTION THREE

a) Form the augmented matrix of the system of equations, convert the matrix to reduced row-echelon form by using equation operations and then describe the solution set of the original system of equations;

$$3x_1 + 2x_2 + 5x_3 = 9$$
$$2x_1 - 3x_2 - x_3 = 6$$
$$4x_1 - 2x_2 + 6x_3 = 16$$

(5 marks)

b) Using vectors prove that "the shortest distance between two points is in a straight line". **(10 marks)**

QUESTION FOUR

a) Evaluate the roots that satisfy the equation;

$$x^2 - 5x - 24 = 0$$ **(3 marks)**

b) Sketch the following graphs;

i. $y = |3|$ **(2 marks)**

ii. $y = -|3|$ **(2 marks)**

iii. $y = |x| - 2$ **(2 marks)**

c) Find a basis for the kernel and the nullity of;

$$\begin{pmatrix} 2 & 4 & 10 & 0 \\ 2 & -2 & -2 & 0 \\ -2 & 0 & 0 & 2 \end{pmatrix}$$ **(6 marks)**

QUESTION FIVE

a) Suppose that $\{v_1, v_2, v_3, \ldots, v_n\}$ is a set of vectors.
Prove that $\{v_1 - v_2, v_2 - v_3, v_3 - v_4, \ldots, v_n - v_1\}$ is a linearly dependent set. **(5 marks)**

b) Evaluate whether the following vectors are linearly independent or dependent;

i. $\left\{\begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 10 \end{bmatrix}\right\}$ **(10 marks)**

ii. $\left\{\begin{bmatrix} 2 \\ 4 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 2 \\ 14 \end{bmatrix}\right\}$

QUESTION SIX

a) Compute for the unknown vector in the following equations; **(3 marks)**

$$7u = 2\begin{bmatrix} -4 \\ 7 \\ 5 \\ 10 \end{bmatrix} + 3\begin{bmatrix} 5 \\ 14 \\ 13 \\ -2 \end{bmatrix}$$

b) Find the reduced row echelon form of the following matrix

$$\begin{pmatrix} 2 & 4 & 6 & -2 \\ 8 & 10 & 12 & 6 \\ 7 & 8 & 9 & 5 \end{pmatrix}$$

(6 marks)

c) Given that $f(x) = 2x + 3$ and $g(x) = x^2 - 1$ compute;

i. $f[g(x)]$ **(3 marks)**

ii. $g[f(x)]$ **(3 marks)**