The Management University of Africa

Sponsored by the Kenya Institute of Management

UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF ARTS IN EDUCATION

MTH212 : DISCRETE MATHEMATICS

DATE: 28th march 2022

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provided.**

QUESTION ONE

a) Without explaining your answer, determine which of the following are statements.

  i. The sun rises from east to west **(1 mark)**

  ii. She can't possibly be twenty, can she? **(1 mark)**

  iii. Every morning is a start of a new day. **(1 mark)**

b) Illustrate the distributive law of logical equivalence **(2 marks)**

c) Given the statement "If I am thirsty, I will drink juice", write a;

  i. Converse statement to it **(1 mark)**

  ii. Inverse statement to it **(1 mark)**

  iii. Contrapositive statement to it **(1 mark)**

d) Let x, y and z be integers. Prove that if $x|y$ and $y|z$ then $x|z$ **(4 marks)**

e) Use truth table to prove the following formulas; **(5 marks)**

$$\neg M \leftrightarrow N \Leftrightarrow (M \to \neg N) \wedge (\neg N \to M)$$

f) Prove or give a counterexample:

"If $\gcd(ab) = d$ and $\gcd(bc) = 1$, then $\gcd(ac) = 1$" **(3 marks)**

g) Formalize the following argument then determine the result using logical inferences. "Either Dan works or Purity does not work. If it is not true that both Purity works and Brandon does not work, then clearly Celina does not work. However Dan does not work." Does Celina work? **(5 marks)**

QUESTION TWO

a) Prove the following biconditional logical equivalence;

$$(M \wedge N) \vee (\neg M \wedge \neg N) \equiv M \leftrightarrow N$$ **(4 marks)**

b) Without using the truth tables, determine whether this is a tautology or a contradiction. *(Hint: Laws of logic)*

$$(M \wedge N) \to (M \vee (N \wedge M))$$ **(7 marks)**

c) Simplify the expression fully; $\frac{4-i^3}{i+i^5}$ **(3 marks)**

d) Write a simple example of a conditional statement **(1 mark)**

QUESTION THREE

a) Prove that if n is even, then n^3 is even **(4 marks)**

b) Using an arbitrary element x for each set, prove the following statement;

$\overline{M \cup N} = \overline{M} \cap \overline{N}$ **(5 marks)**

c) Let m and n be integers. Use contrapositive to prove that if 3 does not divide mn, then 3 does not divide m and 3 does not divide n. **(6 marks)**

QUESTION FOUR

a) Given the following sets $M = \{a, b, c, d, \}$, $N = \{c, d, e, f\}$, $P = \{b, c, g\}$ evaluate the following;

i. $M \cup N$ **(1 mark)**

ii. $N \cap P$ **(1 mark)**

iii. $N \Delta P$ **(1 mark)**

iv. $M \backslash P$ **(1 mark)**

v. $Cardinality\ of\ N$ **(1 mark)**

vi. $Power\ set\ of\ P$ **(2 marks)**

b) A survey was done to help in planning for drinks in a party, and the following data obtained.

- 150 guests took soft drink
- 200 guests took tea
- 115 guests took beer
- 80 guests took soft drink and beer
- 45 guests took beer and tea
- 90 guests took soft drink and tea.
- 30 guests took all the three drinks

How many of the guests took:

i. At least two types of drink **(2 marks)**

ii. Exactly one type of drink **(2 marks)**

iii. Only beer **(2 marks)**

iv. Exactly two types of drinks **(2 marks)**

QUESTION FIVE

a) Prove the following conditional statement;

$$M \wedge N \equiv \neg(N \rightarrow \neg M)$$ **(2 marks)**

b) Prove that if a is rational and b is irrational, then $2a + b$ is irrational **(5 marks)**

c) Let M and N be two events with $P(M) = 0.3$, $P(N) = 0.5$ and $P(M \cap N) = 0.15$.;

i. Determine if M and N are independent events **(2 marks)**

ii. Evaluate $P(M \cup N)$ **(3 marks)**

iii. Compute $P(M' \cup N')$ **(3 marks)**

QUESTION SIX

a) Negate $\exists_x(M_x \wedge N_x)$ **(4 marks)**

b) Negate $\forall_x(M_x \rightarrow N_x)$ **(4 marks)**

c) Prove that if a is an odd integer, then $a^2 - 5$ is divisible by 4 but never 8.

(7 marks)