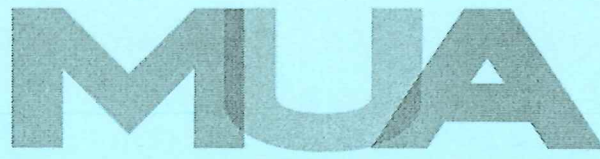**The Management University of Africa** | MUA
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UNDERGRADUATE UNIVERSITY EXAMINATIONS

SCHOOL OF MANAGEMENT AND LEADERSHIP

DEGREE OF BACHELOR OF EDUCATION ARTS

MTH 222 :   CALCULUS II - Integral

DATE: 25TH JULY 2022

DURATION: 2 HOURS

MAXIMUM MARKS: 70

INSTRUCTIONS:

1. Write your registration number on the answer booklet.
2. **DO NOT** write on this question paper.
3. This paper contains **SIX (6)** questions.
4. Question **ONE** is compulsory.
5. Answer any other **THREE** questions.
6. Question **ONE** carries **25 MARKS** and the rest carry **15 MARKS** each.
7. **Write all your answers in the Examination answer booklet provided.**

QUESTION ONE (25 MARKS)

a) The gradient function for a curve is given by $3x^2 + 4x - 3$. If the curve passes through the point $(1, 4)$ evaluate;

    i.   The equation of the curve. (3 marks)

    ii.   The equation of the tangent to the curve at $(1, 4)$ (3 marks)

b) Compute the area bounded by the $x - axis$, the curve $y = 3x^2 + 2x - 2$ and the lines $x = 1$ and $x = 3$ (4 marks)

c) Evaluate $\int \dfrac{x^3 + 4x^5 - 3x^2}{x^3}\, dx$ (3 marks)

d) Compute the value of k if $\int_0^2 (kx^3 - 3x^2)dx = 16$ (4 marks)

e) A particle moves in a straight line from a fixed point. Its velocity $V\ ms^{-1}$ after t seconds is given by $V = 9t^2 - 4t + 1$. Calculate the distance travelled by the particle during the third second. (4 marks)

f) Evaluate $\int x \sin(x^2)\, dx$ (4 marks)

QUESTION TWO

a) Evaluate $\int x \sin x\, dx$ (3 marks)

b) Evaluate $\int x \ln x\, dx$ (4 marks)

c) Consider the region bounded by the graphs of $f(x) = x^2 + 1$ and $g(x) = 3 - x^2$.

d) Write the integral for the volume of the solid of revolution obtained by rotating this region about the $x - axis$. (8 marks)

QUESTION THREE

a) Complete the table below for the function $y = x^2 - 3x + 6$

   in the range $-2 \leq x \leq 8$.   **(2 marks)**

| X	-2	-1	0	1	2	3	4	5	6	7	8
Y											

b) Use the trapezoidal rule with 10 trapezia to estimate the area bounded by the curve, $y = x^2 - 3x + 6$, the lines $x = -2$, $x = 8$, and the $x - axis$ **(4 marks)**

c) Use mid-ordinate rule with 5 strips to estimate the area in (b) above   **(4 marks)**

d) Use integration to find the area   **(5 marks)**

QUESTION FOUR

The diagram below shows a straight line intersecting with the curve at the points P and Q. The line also cuts $x - axis$ at $(7, 0)$ and $y - axis$ at $(0, 7)$.
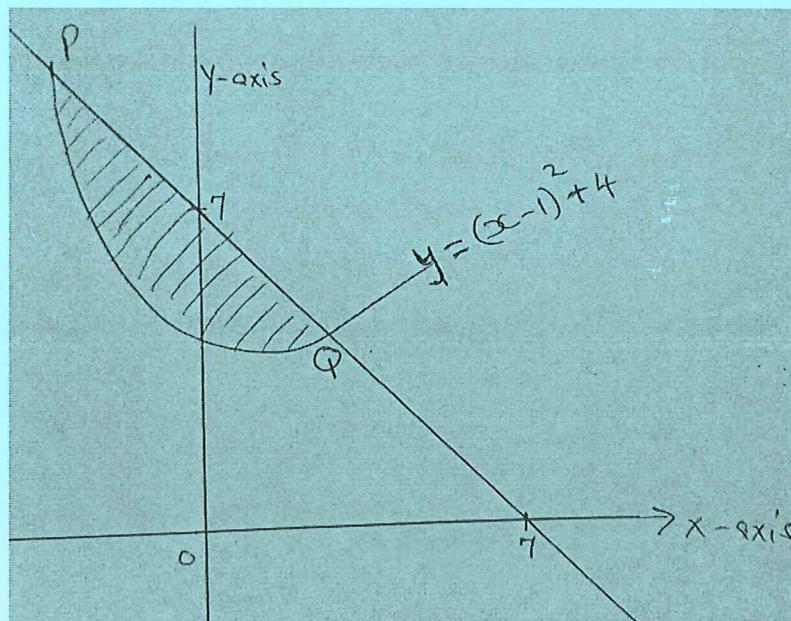
a) Evaluate the equation of the straight line in the form $y = mx + c$ **(2 marks)**

b) Compute the coordinates of P and Q   **(4 marks)**

c) Use integration to calculate the area of the shaded region   **(9 marks)**

QUESTION FIVE

a) Evaluate $\int \sin^{-1} x \, dx$ **(5 marks)**

b) Compute the integral $\int \sin^3 x \cos^2 x \, dx$ **(5 marks)**

c) The region under the curve $y = \sin x$ and above the $x - axis \; for \; 0 \le x \le \pi)$ is revolved around the $y - axis$. Compute the volume **(5 marks)**

QUESTION SIX

a) Compute the integral $\int \dfrac{x+1}{x^2(x-1)} \, dx$ **(8 marks)**

b) A tank contains 200 L of salt water with a concentration of 4g/L. Salt water with a concentration of 3g/L is being pumped into the tank at the rate of 8L/min, and the tank is being emptied at the rate of 8 L/min. Assume the contents of the tank are being mixed thoroughly and continuously. Let S(t) be the amount of salt (measured in grams) in the tank at time t (measured in minutes).

    i.   Compute the amount of salt in the tank at time t=0. **(1 mark)**

    ii.   Evaluate the rate at which salt enters the tank **(2 marks)**

    iii.   Evaluate the rate at which salt leaves the tank at time t **(2 marks)**

    iv.   Compute $\dfrac{dS}{dt}$, the net rate of change of salt in the tank at time t **(2 marks)**